

Solutions to Mathematics Workbook-1 | Quadratic Equations & Inequalities [2022]

Level - 1	Daily Tutorial Sheet 3
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31. $3x - 3 \geq x + 5$ and $\frac{1}{x-1} \geq \frac{1}{8} \Rightarrow 2x - 8 \geq 0$ and $\frac{8 - (x-1)}{8(x-1)} \geq 0$
 $\Rightarrow x \geq 4$ and $\frac{x-9}{8(x-1)} \leq 0 \Rightarrow x \geq 4$ and $x \in [1, 9] \Rightarrow x \in [4, 9]$

32. $f(x) = \frac{(x-3)(x+2)(x+5)}{(x+1)(x-7)}$
Hence, $f(x) > 0 \forall x \in (-5, -2) \cup (-1, 3) \cup (7, \infty)$
 $f(x) < 0 \forall x \in (-\infty, -5) \cup (-2, -1) \cup (3, 7)$

33. $x \in (-\infty, -4] \cup (-2, 1] \cup (3, \infty) \cup \{2\}$

$$\frac{(x-1)(x-2)^2(x+4)}{(x+2)(x-3)} \geq 0$$

($x = 2$ will not be included to make intervals but the inequality is satisfied at $x = 2$)

$$\Rightarrow x \in (-\infty, -4] \cup (-2, 1] \cup (3, \infty) \cup \{2\}$$

34. $\frac{(x-1)(x-2)(x-3)^2}{(x-4)^2(x-5)^3} < 0 \Rightarrow \frac{(x-3)^2}{(x-4)^2(x-5)^2} \left(\frac{(x-1)(x-2)}{(x-5)} \right) < 0, \quad x \neq 3, 4, 5$
 $\Rightarrow x \in (-\infty, 1) \cup (2, 3) \cup (3, 4) \cup (4, 5)$

35.(C) $x^2 + 6x - 27 > 0$ and $x^2 - 3x - 4 < 0 \Rightarrow (x < -9 \text{ or } x > 3)$ and $-1 < x < 4 \Rightarrow 3 < x < 4$

36.(D) Given that, $\frac{x+2}{x^2+1} > \frac{1}{2} \Rightarrow x^2 - 2x - 3 < 0 \Rightarrow (x-3)(x+1) < 0 \Rightarrow -1 < x < 3$

The integral values of x are 0, 1, 2.

$\therefore$ The number of integral solutions is 3.

37.(A) $5x + 2 < 3x + 8 \Rightarrow 2x < 6 \Rightarrow x < 3$

If $x-1 > 0$ then $\frac{(x+2)}{(x-1)} < 4 \Rightarrow x+2 < 4x-4 \Rightarrow 3x > 6 \Rightarrow x > 2$

If $x-1 < 0$ then $\frac{(x+2)}{(x-1)} < 4 \Rightarrow x+2 > 4x-4 \Rightarrow 3x < 6 \Rightarrow x < 2 \quad \therefore \text{Range} = (-\infty, 1) \cup (2, 3)$

38. $\frac{3x-6}{x+2} \geq 0 \Rightarrow x \in (-\infty, -2) \cup [2, \infty)$ and $x^2(x^2 - 5x + 6)(1 - x^2) \geq 0 \Rightarrow x \in [-1, 1] \cup [2, 3]$
From both $\therefore x \in [2, 3]$

$$\Rightarrow x(x+3) + x^2 - 2 \geq 0 \Rightarrow x^2 + 3x + x^2 - 2 \geq 0 \Rightarrow 2x^2 + 4x - x - 2 \geq 0$$

$$\Rightarrow 2x(x+2) - 1(x+2) \geq 0 \Rightarrow (2x-1)(x+2) \geq 0$$

$$\Rightarrow x \in (-\infty, -2] \cup \left[\frac{1}{2}, \infty\right) \dots \dots \text{(ii)}$$

$$(i) \cap (ii) \Rightarrow x \in (-\infty, -3) \dots \dots \text{(A)}$$

Case 2: $-3 \leq x < 0$

$$-x(x+3) + x^2 - 2 \geq 0 \dots \dots \text{(iii)}$$

$$\Rightarrow -x^2 - 3x + x^2 - 2 \geq 0 \Rightarrow 3x + 2 \leq 0$$

$$\Rightarrow x \leq \frac{-2}{3} \dots \dots \text{(iv)}$$

$$(iii) \cap (iv) \Rightarrow x \in \left[-3, -\frac{2}{3}\right] \dots \dots \text{(B)}$$

Case 3: $x \geq 0 \dots \dots \text{(v)}$

$$x^2 + 3x + x^2 - 2 \geq 0 \Rightarrow x \in \left[\frac{1}{2}, \infty\right) \dots \dots \text{(C)}$$

$$(A) \cup (B) \cup (C) \Rightarrow x \in \left(-\infty, -\frac{2}{3}\right] \cup \left[\frac{1}{2}, \infty\right)$$

45.(D) $\left| |x| - 1 \right| < |1 - x|, x \in R$

+ve +ve

Squaring both sides,

$$\Rightarrow (|x| - 1)^2 < (1 - x)^2 \Rightarrow x^2 + 1 - 2|x| < 1 + x^2 - 2x$$

$$\Rightarrow |x| > x \Rightarrow x < 0 \Rightarrow x \in (-\infty, 0)$$

Alternate method 1:

Case - I: $x \geq 0$ LHS is $|x - 1|$ and RHS is $|1 - x|$

$|t| = |-t| \forall t \in R$ and thus LHS = RHS and the inequality fails.

Case - II: $x < 0$ i.e. $(-x) > 0$

LHS: $|-x - 1|$

RHS is $|-x + 1| = (-x) + 1$ $\{-x$ being positive $\}$

$|(-x) - 1| < (-x) + 1$ will always hold.

$$\Rightarrow x \in (-\infty, 0)$$

Alternate method 2:

Critical points are $x = 1$ for $|1 - x|$

$x = 0$ for $|x|$ and $x = \pm 1$ for $||x| - 1|$

Case-I: $x \geq 1$

$$x - 1 < x - 1$$

Impossible

Case - II: $x \in [0, 1)$

$$1 - x < 1 - x$$

impossible

Case – III: $x \in [-1, 0)$

$$x + 1 < 1 - x$$

$$\Rightarrow x < 0$$

Hence, $x \in [-1, 0)$

Case – IV: $x < -1$

$$-x - 1 < 1 - x$$

$$\Rightarrow -1 < 1$$

True always.

Hence, $x < -1$

Combining all cases $x \in (-\infty, 0)$.